THEORY OF LIGHTNING

B. N. Kozlov

UDC 537.521.7

1. The Mechanism by Which Lightning Develops. The development of lightning is an oscillatory relaxation process which is basically electromagnetic in nature, common to both linear and ball forms of lightning [1, 2]. Each oscillatory cycle of the lightning consists of stages of "electromagnetic activity" and a subsequent stage of "electromagnetic calm." The process is related to a charged channel — a column of air emerging from a cloud, which is breaking up in the atmosphere, separated from the surrounding medium of ionized air and of reduced density. The charged channel of lightning, in view of the oscillatory mode of the process, periodically transfers from a state of a weakly ionized almost nonconducting charged path into a good conducting column and vice versa. Along the lightning channel, in the same way as along a single-conductor waveguide line of the rod antenna type, from the cloud to the head of the channel a particular kind of aperiodic electromagnetic wave (a channel wave) travels governing the transfer of energy and charge of the lightning. The propagation of the channel wave is extremely nonlinear and nonequilibrial, since these waves are characterized by the ability to create a nonequilibrium high electrical conductivity in the channel behind the wave front, necessary for its propagation, which depends on the wave field, by the ionizing action of the wave field. The channel wave arises at the beginning of each cycle at the point where the axis of the channel intersects the surface of the cloud, after which it propagates hemispherically in space outside the cloud with a speed close to the velocity of light (determined by the solution of Maxwell's equations). In the cloud, while the channel wave is propagating, due to streamer discharges from the surfaces of ice crystals and drops of water, high electrical conductivity rapidly occurs, as a result of which the cloud becomes a good conductor. This fact, i.e. the high conductivity of the cloud during the lightning discharge, derived theoretically in a separate paper, in this case can be regarded as experimental, since the energy of the lightning close to the initial electrical energy of the cloud [3] can be obtained from the cloud only when the cloud has high electrical conductivity. The energy of the lightning is transferred by the channel waves mainly outside the lightning channel in the form of electromagnetic-field energy and is only absorbed in the volume of the channel. The electromagnetic field of the channel wave, propagating in air outside the lightning channel with the velocity of light, penetrates into the volume of the channel from the sides by diffraction. Hence, the channel wavefront moves along the channel outside it and along its surface, and also inside the channel at least in a thin surface layer, with the velocity of light. The field of the channel wave in the channel produces a macroscopic (volume-distributed) ensemble of electron avalanches which occur from the initial electrons or ions comprising the "bare" ionization of the channel. The intense electron-avalanche process occurring almost simultaneously immediately behind the wave-front leads to almost ideal electrical conductivity of the part of the channel behind the wavefront [2], while the extremely small electrical conductivity of the parts of the channel in front of the wavefront where there is no field is unconnected with the Joule losses. Hence, the channel waves propagate along initially weakly ionized almost nonconducting channels in the same way as along good-conductor waveguide lines. The more high-powered the channel wave the greater the electrical conductivity connected with it by the field behind the wavefront, and the closer the propagation is to ideal propagation. Under lightning conditions the channel waves supplied with very high potentials, propagate practically ideally with the velocity of light, the extension of the lightning channel being unconnected with the considerable transfer of energy and charge of the lightning, and occurs abruptly with the velocity of longitudinal electrical drift of the charges, periodically introduced into the channel by the channel electromagnetic waves propagating with electromagnetic velocity. The process of propagation of the lightning is therefore a two-velocity process.

In the regions outside the channel, in particular in the extension of the channel along its axis, the propagating field cannot produce considerable electrical conductivity since outside the channel there is insufficient bare charge density. It is also important that in the lightning channel, (particularly the strongly heated region on the axis of the channel) electron capture, in view of the high temperature, should not occur, and the excess negative charges exist in the form of free electrons. The increased temperature of the channel and the reduced density of the air in it lead to improved conditions for current flow in the channel (in the axial core of the channel) compared with the shell (the remaining part of the channel) and correspondingly to an axial current concentration leading to additional heating of the channel, etc.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 82-104, March-April, 1979. Original article submitted June 30, 1977.

0021-8944/79/2002-0187$07.50 ©1979 Plenum Publishing Corporation

When it reaches the head of the channel, the channel wave is reflected, since the geometrical extension of the channel is a region where, in view of the absence of considerable bare ionization, there may not be high electrical conductivity, and this region therefore behaves as a dielectric. The channel wave is reflected so that from the head of the channel, as from the end of a cutoff single-conductor waveguide line, part of the energy is lost as radiation. The reflected wave is usually almost immediately absorbed in the leading part of the channel, where the resistance of the channel is particularly high. The absorption of the wave energy primarily in the leading part of the channel to a certain extent is analogous to the well-known fact of the absorption of the energy of an elastic wave at the end of a converging filament (the "cowboy whip effect"). In view of the gradual increase in the resistance in the leading part of the channel and the "blurring" of the wavefront the intensity of the reflected and radiated waves is comparatively small. (As is well known, at the blurred boundary of the media and in the case of blurred wavefronts, the waves may decay generally without reflection and radiation.)

With the attenuation of the channel electromagnetic wave the stage of electromagnetic activity is completed in each cycle, and the stage of electromagnetic calm begins in the same cycle. After the attenuation of the channel wave the channel remains charged up to the potential of the cloud and has high electrical conductivity everywhere, except the part at the boundary with the cloud. In the part close to the cloud the electrical conductivity drops rapidly due to the very intense recombination and capture processes, since the electric intensity on the surface of the channel in this part is less than the critical value E^*, necessary to maintain high electrical conductivity. The channel at the instant of transition from the active to the calm state is therefore disconnected from the cloud and ceases to obtain energy and charge from it. The electric intensity on the surface of the leading part of the channel at the beginning of the calm period is extremely large (the "sharp-point effect") and much greater than the intensity of the external field (produced by the charges of the remaining parts of the channel and the charges of the cloud). The external field (that is, outside the head of the channel) is not therefore in the initial phase of the calm stage of the directing action on the drift of the charges of the leading part of the channel, i.e., in the extension of the channel. The drift motion of the leading part of the channel in these initial phases of the calm stages is determined by the inherent field of the charges of the head of the channel, as a result of which it is unstable and chaotic. Any random perturbation changes the direction of motion of the head of the channel, and the high electrical conductivity of the channel ensures the development of instabilities by the influx of energy. At these instants, in addition to chaotic wandering of the head of the channel branching of the channel due to instability is possible. The head of the channel, due to the sharply increased concentration of the field and energy dissipation [2], glows in this phase particularly brightly, and its velocity, constantly changing direction, has the highest absolute value. The unordered motions of the head of the channel in the initial phase of the calm stage of each cycle are unrelated to the systematic displacement of the head of the channel in any specified direction, for example, along the external field, which the head of the channel at these instants does not experience in view of its smallness compared with the inherent field of the charges of the head of the channel. The head of the lightning channel at this time wanders randomly inside a volume, forming distinctive junctions of the trajectory of the head, and on average remains on the spot although it moves rapidly.

The omnidirectional drift spreading out of the channel charges leads to a reduction in the electric field at the surface of the channel. When this field strength becomes less than a critical value E^* everywhere on the surface of the channel, ionization ceases, and intense relaxation recombination and capture processes rapidly reduce (in a time of the order of 10^{-8}–10^{-10} sec) the nonequilibrium high conductivity in the channel. The drift spreading out of the charges in the channel when there is no high electrical conductivity leads to a further reduction in the field strength on its surface, in particular, in the region of the head. The external field becomes considerable compared with the natural field of the charges in the channel head. Under these conditions the field which is external to the charges in the channel head directs the drift motion of the charges of the head, and this motion becomes ordered and not random. Beginning from this instant the second phase of the calm stage begins, connected with the systematic progress of the head of the discharge. With the relaxational reduction in the potential of the channel due to spreading out of the charges when the electrical conductivity of the channel is low, particularly at the boundary with the cloud, the potential difference between the cloud and the channel and the field at the initial point of the channel (the point where the axis of the channel intersects the surface of the cloud) increase. At the instant when the intensity at the initial point on the surface of one of the crystals or drops of the cloud reaches the critical value required for a channel wave to occur (the electromagnetic wave of spark breakdown) at the initial point the next channel wave occurs, which corresponds to the beginning of a new pulsation cycle.

At each cycle the channel is again regenerated and extended by a certain amount due to the longitudinal drift of the charges acted upon by the field. The self-propagating process of the development of the lightning may occur from any prolonged weakly ionized region of small dimensions (the nucleus of the lightning), if this region is in air and is connected with a cloud having a fairly high potential. Channel waves are usually produced on sharp needle-shaped crystals which exist in storm clouds [4], since the temperature of the storm clouds is negative [5-7].

When making observations from large distances the structure of the junction points, i.e., the regions of random wandering of the channel head, is usually not seen, and the junction points are observed as wider and brighter parts of the channel with possible breaks in the trajectory or branching. Since only rapid random motion of the head of the discharge occurs at the junction points without any systematic progress of the discharge, in observations the junction points are perceived as "interruptions." Thus, "interruptions of the multistage leader" of the lightning are in fact phases of particularly rapid motion, which, although random, do not lead to a constant systematic displacement of the discharge. The channel with clearly visible junction points of considerable dimensions is perceived as "beaded lightning" [3]. A more detailed consideration enables one to observe very rapid random motion of the head of the channel at each junction point with very bright illumination. For small velocities of motion of the head of the channel, when the directed displacement of the head of the channel during a single cycle is small, the sequence of junction points which occur rapidly one after another, is perceived from considerable distances as a single moving junction point, which can be identified with ball lightning of large dimensions. The usual ball lightning is a displaced region of illumination of the channel head with a weak very slowed down version of the lightning process when the channel as a whole is invisible [1, 2].

By definition [2] the channel is a charge-current region of the channel electromagnetic wave, i.e., the part of the channel behind the wavefront where the field of the wave, propagating with the velocity of light, produces high electrical conductivity, which depends on the electric field strength, and high current concentrations and uncompensated space charges occur due to the action of the field. The vertex of the channel is at the point of intersection of the channel axis and the front of the channel wave and moves along the axis of the channel with the velocity of light. The channel, as an element of the channel electromagnetic wave, is essentially an electromagnetic formation, always tangent to the electromagnetic wavefront at its vertex. This is the main difference between the channels and streamers, which are usually described by the equations of electrostatics, i.e., they are electrostatic formations and propagate with a velocity much less than the velocity of light [8-10]. Spark discharges on a laboratory scale are accurately described by streamer theory [8-10]. This theory, confirmed theoretically and experimentally in spark gaps in the laboratory, is not applicable, however, to lightning discharges [2]. Under lightning conditions the electromagnetic mechanism of relaxation oscillations and the propagation of energy, charge, and current fronts with the velocity of electromagnetic waves as the channel of the discharge progresses with the drift velocity, i.e., a two-velocity mode [1, 2], is decisive. According to the relaxation picture, the vertex of the channel moves from the source (cloud) to the vertex of the discharge, after which the channel disappears, and after a certain time the next channel propagates from the source to the head, etc. According to streamer theory lightning is the propagation of a single streamer, progressing with a velocity much less than the velocity of light. According to the relaxation theory the lightning process is determined by successive transits from the cloud to the head of the discharge, which moves with the drift velocity, of many channels (of the order of hundreds), each of which propagates with the velocity of light.

The relaxation oscillatory mechanism of the discharge acts over kilometer lengths of the discharge. The relaxation theory is constructed asymptotically, i.e., for conditions far from the streamer-relaxation boundary, like the streamer theory [8-10], it is constructed asymptotically for conditions far from the avalanche-streamer transition. The transition regions are extremely complex for calculations. The asymptotic description far from the transition region corresponds quite well to the actual conditions of the lightning. But the asymptotic description which relates to the conditions far from the transition region between the streamer and relaxation mechanisms, does not determine the parameters of the transition region. One of the main manifestations of the relaxation mechanism of a spark discharge is the gradual nature of the propagation of the lightning. The smallest observed length of the pulsation stage of the lightning is 10 m [3], so that the relaxation mechanism occurs for discharge lengths greater than this value. Lightning is never shorter than several hundred meters [11]. Hence, the boundary between the streamer and relaxation mechanisms of the spark discharge in atmospheric air corresponds to a discharge length of hundreds of meters. The electromatic-relaxation picture of a lightning discharge agrees quite well with observational data. Regularly repeating propagation of channel waves from the cloud to the head of the channel with the velocity of light has been recorded in observations as transient flashes of a lightning channel — flickers of lightning [3, 8, 12], and in the case of infralightning, when the channel as a whole is invisible, it has been observed in the form of flashes of the head of the channel, i.e., ball lightning [13, 7, 14]. From observations of linear lightning during the flashes "bright illumination encompasses all the stages" [11], in which the passage of a channel in each cycle along the whole channel from the cloud to the head occurs. An extremely important fact is the existence of lightning emerging from the cloud but not reaching the earth [3, 8, 12, 5] which arises from relaxation theory and which has been observed experimentally. (According to the streamer theory of a lightning discharge emerging from a cloud, it should reach the earth.) The characteristic lengths of lightning (5 km) and its maximum length (200 km) predicted by the relaxation theory agrees with the results obtained in [3, 5, 11, 12, 15] (according to the streamer theory the length of the lightning should be 0.2 km [10]). The effect of the increased brightness of the head of the lightning channel which follows from the relaxation theory has been recorded in observations [8] and appears in all cases of ball lightning [7, 14]. The lightning process (in the ball-lightning version) has been seen most clearly and in greatest detail on photographs [13], a description of which is given in [16]. The photographs clearly show the meandering nature of the trajectory inside the junction points, and the increased brightness of the junction points can be seen. This has also been observed in the photographs taken by Deryugin [14]. The approximate agreement between the minimum energy dissipation of linear lightning and the maximum energy dissipation of ball lightning predicted by the relaxation theory has been confirmed experimentally [7, 2, 17].

An accurate mathematical description of the channel process, i.e., the relaxation electromagnetic oscillatory process, determining the development of the lightning, requires the solution of a nonstationary (with a conductivity of the medium varying with time), two-dimensional (axisymmetrical) nonlinear (with the electrical conductivity of the medium depending on the field of the wave) and nonequilibrium (the ionization of the medium is not determined by the temperature) problem of the propagation of an electromagnetic field in a nonuniform medium taking into account the gas-dynamic motion and thermal conductivity. These features of the process are decisive properties and cannot be ignored in order to simplify the calculations. The extreme complexity of the process requires a special simplified approach to its mathematical description. Such an approach is possible due to the particular features of the process itself. Its main feature is its cyclical nature. The important parameters of the state of the system (the potential of the cloud, the temperature and density of the air in the channel, and the length of the channel) do not change very much during a single cycle and can be assumed constant within the cycle and considered to change abruptly from cycle to cycle. Each cycle moreover decays in the electromagnetically active and calm stages, and each of the stages can be described by its own equations taking into account its fundamental features. Thus, although all the above-mentioned features of the process are important, and are not amenable to simultaneous consideration in view of the considerable mathematical complexity, the approach formulated enables them all to be taken into account fairly effectively. The asymptotic nature of the theory connected with the very high values of the supplying potentials leading to pronounced electromagnetic properties of the process is also a simplifying factor. In this case equations are used which do not describe the transitions between the electrodynamic and electrostatic regions, in the same way as the ultrarelativistic approximation does not describe the transition to nonrelativistic relations. Hence the equations of relaxation theory, being based on the electromagnetic picture of the phenomenon, do not transfer into the equations of streamer theory [8-10], which are based on the equations of electrostatics. These are opposite limiting cases.

When there is a charged channel (in its initial state a weakly ionized charged channel [2]), the channel waves can propagate in the complete absence of an external field. The self-propagating process of the development of the lightning, occurring without a previously constructed channel, is possible when there is a weak external field, which has no effect on the propagation of the channel waves and only advances the head of the discharge channel in a certain direction. An example of this is a lightning discharge from a cloud which has two plane (parallel to one another) electrically charged layers, one of which (the N-layer) is negative and has high electrical conductivity, and the second (the p-layer) which is situated below it, positive and not electrically conducting. The charges of the layer, generally speaking, are not equal. Between the layers (from the negative conducting to the positive nonconducting) a lightning discharge develops directed downwards. On reaching the positive layer and penetrating it, the discharge continues its propagation into the external region, if there is at least a small incipient channel there in the form of an initial weakly ionized charged channel. The channel wave, on reaching its end, dies away after reflection and partial radiation, leaving the channel highly electrified. If there is a weak external field (less than 3 MV/m), due to the small difference in charges of the layers or the charges of other clouds, the channel will extend by directional drift, periodically acquiring energy and charge due to the regularly passing channel waves. Hence, under conditions when the dissipative resistance of the core of the channel, i.e., its inner part from the point where it arises to emergence from the positive layer, is small (compared with the wave impedance of the channel), the potential difference between the main column of the channel (its external part) and the positive layer will be equal to the potential difference between the layers.

In the ideal case, for mathematical simplicity, the electrical conductivity of the N-layer will be assumed to be infinite, and the distance between the layers will be assumed to be infinitely small. In this case the cloud becomes a double electrical layer with infinite electrical conductivity having, generally speaking, a certain uncompensated charge per unit surface, due to the difference between the charges of the layers. The field produced by this cloud in external space can be extremely small, whereas the potential difference supplying the discharge may be very high. From the calculation point of view this simplest model is intended primarily to explain the main features of the channel waves and only describes very approximately the actual situation in the atmosphere, where the N-zone and p-zone of the cloud have larger dimensions in all directions [3]. For more detailed calculations we will use another model below in which the absence of the important effect of the external field on the propagation of the channel waves is not so clearly seen. Physically this becomes obvious when the channel waves are compared with electromagnetic waves in lines, for example, in a cable. The field of the charges in the source (the generator) has no effect on the wave propagation, and only the potential difference is important. In the case of a single-conductor line this is the potential difference between the line and earth produced by the generator, and if the line is directed vertically, is very similar to the channel emerging from a cloud, with the difference that the waves themselves propagating along it produce high electrical conductivity in the channel.

2. Transfer of the Energy of the Lightning. The propagation of the channel waves, which carry the energy of the lightning, is described by Maxwell's equations

$$\mathrm{rot}\,\mathbf{E} + \mu_0 \partial \mathbf{H}/\partial t = 0,\ \mathrm{rot}\,\mathbf{H} - \varepsilon_0 \partial \mathbf{E}/\partial t = \mathbf{J}, \qquad (2.1)$$
$$\varepsilon_0\,\mathrm{div}\,\mathbf{E} = \rho,\ \mathrm{div}\,\mathbf{H} = 0,$$

where for air we take $\varepsilon = \varepsilon_0$, $\mu = \mu_0$.

We will consider the limiting channel wave using an example in which the pattern of the field distribution of the channel waves in space can be seen most simply. By a limiting channel wave we mean a channel wave the field strength of which is so large that the electrical conductivity produced by the field of the wave in the channel behind the wavefront leads to a dissipative resistance (determining the Joule losses), which is negligibly small compared with the wave impedance of the channel. Under these conditions the energy losses are negligibly small and propagation occurs almost ideally. The channel in its initial state is a weakly ionized charged channel, but behind the wavefront it becomes conducting. (Note that the usual concentration of ions in the air is negligibly small compared with the initial air concentration in the charged channel.) In the formulation considered the pattern reduces to the propagation of a multistage electromagnetic wave along the channel like an ideally conducting single-conductor line. The problem is solved in spherical coordinates (x, θ, φ) in the following formulation. We wish to determine the field of the electromagnetic wave in the conical region between an ideally conducting plane $\theta > \theta_{00} = \pi/2$ (simulating the surface of the cloud), and a thin ideally conducting cone $\theta = \theta_0 \ll 1$ (simulating the surface of the part of the channel behind the wavefront). The wave occurs at the instant $t = 0$ when a potential difference $V_0(t)$ is connected in the infinitely small gap between the cone and the plane at the point $\mathbf{x} = 0$. The solution is considered for a finite time interval from the initial instant $t = 0$ when the wave arises to the instant t_0 when the wavefront reaches the radius $x = a_0$, outside the limits of which there is no channel. The boundary conditions are that the tangential component of the electric field on the surfaces $\theta = \theta_{00}$, $\theta = \theta_0$ should vanish. There are no space charges and currents in the region in which the field is determined, i.e., when $0 \le x \le a_0$, $\theta_0 < \theta < \theta_{00}$. The source condition is formulated by assigning a monotonically varying voltage $V_0(t)$ (in the form of a power function) at the ends of the infinitely small part between the cone $\theta = \theta_0$ and the plane $\theta = \theta_{00}$ at the point $x = 0$

$$\lim \int_{\theta_0}^{\theta_{00}} E_\theta x\, d\theta = V_0(t) = \begin{cases} 0, & t < 0, \\ V_{00} t^p, & t \geqslant 0, \quad p \geqslant 0. \end{cases} \qquad (2.2)$$

The solution of Maxwell's equations will be sought in the form

$$\mathbf{E} = -\nabla\Phi - \partial \mathbf{A}/\partial t, \quad \mathbf{H} = \mu_0^{-1}\,\mathrm{rot}\,\mathbf{A}, \qquad (2.3)$$
$$\mathbf{A} = \Psi \mathbf{x}/x, \quad \Phi = F(x, t)\,\Lambda(\theta), \quad \Psi = G(x, t)\,\Lambda(\theta), \quad c_0 = 1/\sqrt{\varepsilon_0 \mu_0}.$$

The surface charge and current densities ω and j on the surfaces $\theta = \theta_0$, $\theta = \theta_{00}$ can be found using the well-known boundary relations, after which the charge Q on the cone is obtained, referred to unit length of the circle radius x, and the radial currents I on the cone $Q = 2\pi x \varepsilon_0 E_\theta(\mathbf{x}, \theta_0, t) \sin\theta_0$, $I = 2\pi x H_\varphi(x, \theta_0, t) \sin\theta_0$.

The functions F and G in (2.3) can be expressed in terms of Q and I as follows:

$$F = \Lambda_0 Q/2\pi\varepsilon_0 \Lambda(\theta_0), \quad G = \Lambda_0 I/2\pi\varepsilon_0 \Lambda(\theta_0), \quad \Lambda_0^{-1} = (d\Lambda/\Lambda d\theta)_0 \sin\theta_0. \qquad (2.4)$$

The following equations are obtained for Q and I:

$$K\partial Q/\partial x + L\partial I/\partial t + E = 0,\ \partial I/\partial x + \partial Q/\partial t = 0, \tag{2.5}$$

where $E = E_x$ is the longitudinal component of the electric field, and the parameters K and L are given by

(2.6)

Substituting (2.3) into Maxwell's equation (2.1) for $E_x = 0$ leads to the angular function $\Lambda(\theta)$, and to the quantity Λ_0 in (2.4) which is given by

$$\Lambda = \ln \operatorname{ctg} \theta/2,\ \Lambda_0 = \ln \operatorname{ctg} \theta_0/2. \tag{2.7}$$

The length of the channel a_0 is related to the radius b_0 of its greatest transverse cross section by the equation $b_0 = a_0 \tan \theta_0$. For a thin channel $\theta_0 \ll 1$

$$\Lambda_0 = \ln 2\nu_0,\ \nu_0 = a_0/b_0. \tag{2.8}$$

For ideal conductivity of the channel the longitudinal component E of the electric field is zero, and for high but finite conductivity, the longitudinal component $E = E_x$ is much less than the transverse component E_θ. In the region outside the channel, where there is no conductivity, the longitudinal component can be neglected in view of its smallness. In the channel behind the wavefront, where the electrical conductivity is high, even a small longitudinal component is important, since it produces a considerable current. By Ohm's law we have

$$E = IR, \tag{2.9}$$

where R is the resistance of unit length of the channel. Equations (2.5) and (2.9) are the well-known "telegraph" equations [18, 19], which are derived here together with the concrete expressions for the parameters K and L for channel waves directly from Maxwell's equations.

Since we are considering a wave which occurs at the initial instant of time $t = 0$ at the point $x = 0$, while the velocity of propagation of the perturbations is finite, a wavefront will exist moving with a certain velocity c. On the front $x = x_0(t)$, by the definition of the front, and for the charge Q per unit length of the channel and current I in the channel we have the following conditions:

$$Q(x_0, t) = 0,\ I(x_0, t) = 0. \tag{2.10}$$

The solution of (2.5) and (2.9) for $R = 0$ for the conditions (2.2) and (2.10) will be sought in the self-similar form

$$\begin{gathered} Q = CV_0(t)f(\xi)/K,\ I = CV_0(t)g(\xi)/Z, \\ V_0 = V_{00}t^p,\ \xi = x/x_0(t),\ x_0 = ct,\ c = \gamma c_0,\ Z = \beta Z_0, \\ Z_0 = \sqrt{KL}, \end{gathered} \tag{2.11}$$

where x_0 and c are the radius and velocity of the wavefront respectively, and C, β, γ are undetermined constants. Substituting (2.11) into (2.5) and (2.9) for $R = 0$ we obtain

$$\begin{gathered} (1 - \gamma^2\xi^2)df/d\xi + p\gamma^2\xi f + p\gamma g/\beta = 0, \\ (1 - \gamma^2\xi^2)dg/d\xi + p\gamma^2\xi g + p\beta\gamma f = 0. \end{gathered} \tag{2.12}$$

For the functions $f(\xi)$, $g(\xi)$, defined by (2.11) we will take the normalization $f(0) = 1$ and $g(0) = 1$, apart from a constant factor. From (2.10) and (2.11) we have $f(1) = 0$ and $g(1) = 0$. Hence, for the system (2.12) we introduce the conditions

$$f(0) = 1,\ g(0) = 1,\ f(1) = 0,\ g(1) = 0. \tag{2.13}$$

The solution of (2.12) for conditions (2.13) determines the parameters $\beta = 1$, $\gamma = 1$ and has the form

$$f(\xi) = (1 - \xi)^p,\ g(\xi) = (1 - \xi)^p. \tag{2.14}$$

Since according to (2.11) the velocity of the front is $c = \gamma c_0$, the front propagates with the velocity of light.

The solution of (2.12) with the conditions (2.13) for $p = 0$ is defined as the limit of the series of solutions (2.14) as $p \to 0$. The solution for $p = 0$ therefore has the form

$$f(\xi) = \begin{cases} 1, & 0 \leqslant \xi < 1, \\ 0, & \xi = 1, \end{cases} \quad g(\xi) = \begin{cases} 1, & 0 \leqslant \xi < 1, \\ 0, & \xi = 1. \end{cases} \tag{2.15}$$

Note that the limiting transition $p \to 0$ in (2.14) reveals the physical meaning of the solution (2.15), the direct determination of which from (2.12) and (2.13) is not completely obvious.

The voltage in the channel is given by the equation

$$V = \int_{\theta_0}^{\theta_{00}} E_\theta x d\theta. \tag{2.16}$$

It follows from (2.16), (2.3), (2.4), (2.6), (2.7), and (2.11), that $V(x, t) = CV_0f$. Nevertheless by (2.16), and (2.2) $V(0, t) = V_0(t)$, and by (2.13) $f(0) = 1$. Hence, $C = 1$ and $V(x, t) = V_0(t)f(\xi)$. Comparing this with the expression for Q from (2.11) we obtain

$$V = KQ. \tag{2.17}$$

There are no Joule losses in the channel in the case considered. The ratio of the voltage $V_0(t) = V(0, t)$ at the input to the channel to the input current $I_0(t) = I(0, t)$ for loss-free propagation is, by definition, the wave impedance of the channel as a waveguide line. From (2.11), (2.3), and (2.6) putting $C = 1$, $\beta = 1$, $g(0) = 1$, we obtain the following expression for the wave impedance:

$$Z_0 = \Lambda_0/2\pi\varepsilon_0 c_0. \tag{2.18}$$

Using (2.11), (2.14), and (2.2) and putting $C = 1$, $\beta = 1$, $\gamma = 1$, we obtain

$$Q = (1 - x/c_0t)^p V_0(t)/K,\ I = (1 - x/c_0t)^p V_0(t)/Z_0. \tag{2.19}$$

It can be seen from (2.19), (2.6), and (2.18) that in the case of an ideal wave considered (an extremely high-power wave producing ideal electrical conductivity behind its front) the current I in the channel and the current Q per unit length of the channel are connected by the equation $I = c_0Q$, where c_0 is the velocity of light. It can also be seen from (2.19) that the charge and current fronts propagate with the velocity of light and not with the drift velocity, although the charges move with the drift velocity. Under conditions when the dissipative resistance of the channel (the resistance determining the Joule losses) is much less than the wave impedance (2.18) (as occurs in the case of lightning), propagation occurs almost ideally and, with a constant feeding potential, is characterized by a profile very close to the rectangular step profile of (2.15).

Using (2.3), (2.4), (2.19), and (2.7) we obtained the desired solution of Maxwell's equations describing the propagation of the channel waves

$$\mathbf{E} = Q(x, t)\mathbf{n}_\theta/2\pi\varepsilon_0 x \sin\theta,\ \mathbf{H} = I(x, t)\mathbf{n}_\varphi/2\pi x \sin\theta, \tag{2.20}$$

where the region in which the field is defined is $0 \le x \le x_0(t)$, $\theta_0 \le \theta \le \theta_{00}$, $0 \le \varphi \le 2\pi$, the vectors $\mathbf{n}_\theta$, $\mathbf{n}_\varphi$ are the unit vectors of the spherical system of coordinates x, θ, φ, while the functions Q(x, t), I(x, t) are given by (2.19). The field inside the channel $0 \le x \le x_0(t)$, $0 \le \theta \le \theta_0$ for ideal propagation is zero, but on the surface of the channel the value of field strength of the wave reaches its highest values given by (2.20). The energy density of the field $u = \varepsilon_0E^2 2 + \mu_0H^2 2$ and the value of the energy flux density vector $\Pi = [EH]$ according to (2.20) outside the channel is given by

$$u = Q^2/4\pi^2\varepsilon_0 x^2 \sin^2\theta,\ \Pi = I^2/4\pi^2\varepsilon_0 c_0 x^2 \sin^2\theta \tag{2.21}$$

and is zero inside the channel. The total energy of the field of the wave is obtained by integrating the energy density u over the volume. For the energy in unit interval of length x we obtain from (2.21) and (2.20) the expression

$$U = KQ^2/2 + LI^2/2. \tag{2.22}$$

Hence, taking (2.17) into account, it can be seen that K and L are the inverse capacitance per unit length of the channel and the self inductance per unit length of the channel. Equation (2.22) also follows directly from Eqs. (2.5). According to (2.21) the energy of the channel wave distributed in the space outside the channel, in the case of thin lightning channels, which they in fact are, is concentrated mainly in the surface of the channel, which plays the role of a directing waveguide line. When $\theta_0 \ll 1$ it follows from (2.20), (2.17), (2.6), (2.11), (2.15), and (2.8) that on the surface of the channel $\theta = \theta_0$ the electric field is given by $E_1 = \nu_0 V_0/\Lambda_0 x$, and in the greatest transverse cross section $x = x_0$, where the field E_1 is a minimum, it is given by $E_0 = \nu V_0/\Lambda_0 x_0$. For $V_0 = 10^8$ V [3, 12], $\nu_0 \simeq 10^3$, $\Lambda_0 \simeq 7.6$, and $x_0 \simeq 10^3$ m (which, as we have seen, is characteristic for lightning), we have $E_0 \simeq 1.3 \cdot 10^7$ V/m. The characteristic strength on the surface of the channel is greater than this minimum value. The channel waves, carrying much more intense electric fields, produce in the channels behind their fronts very high electrical conductivity and therefore propagate almost ideally up to the head of the channel.

3. The Power of the Lightning Taking Losses into Account.

The propagation of channel waves without loss of energy was considered above by solving Maxwell's equations. To describe channel waves taking energy loss into account we will solve Eqs. (2.5) and (2.9). The channel will be assumed for simplicity to be a confined axisymmetrical surface, varying in the same way with time and described in cylindrical coordinates r, φ, x by the equation

$$r = r[x, x_0(t), r_0(t)], \tag{3.1}$$

in which the parameters $x_0(t)$ and $r_0(t)$, which depend on time, are the length and radius of the greatest transverse cross section of the channel respectively. In view of the similarity we have

$$\nu_0 = x_0/r_0 = a_0/b_0, \tag{3.2}$$

where a_0 is the maximum length of the channel (in the given cycle), and b_0 is the radius of the greatest transverse cross section of the channel at the instant t_0, when the vertex of the channel $x = x_0(t)$ reaches $x = a_0$. The channel in the last instant when it exists $t = t_0$, when its vertex reaches the outer limits of the channel, becomes a regenerated channel, whereas from the previous channel to this instant practically nothing remains. Because of intense recombination, the air outside the new channel (which appears at the end of the active period of each cycle) is ionized very little compared with the air in the volume of the newly created channel. In the case of good conducting considerably elongated channels, which they actually are in practice, the parameters K and L in Eqs. (2.5) and the wave impedance Z_0 do not depend on the specific shape of the channel, and are expressed with the condition (3.2) by Eqs. (2.6) and (2.18), while the potential V is related to the charge Q per unit length of the channel by Eq. (2.17), which also follows from the meaning of the quantities occurring in it. It is important to note the nonadditivity of the local resistance R per unit length of the channel occurring in (2.9). The integral of R taken over the whole length of the channel, has no physical meaning and can be infinite under practical conditions. In view of the nonconstancy of the current I over the length of the channel the integral of $EI = RI^2$, expressing the power of the Joule losses taken over the length of the channel has physical meaning. The ratio of this integral to the square of the input current $I_0(t) = I(0, t)$ can be called the dissipative or Joule resistance Z_1, a small value of which compared with the wave impedance (2.18) means that the propagation is close to ideal.

The potential of the channel $V(x, t)$ on the basis of $x = 0$, i.e., at the point of contact with the cloud, is equal to the potential of the cloud $V_0(t)$. The potential of the cloud V_0, which varies only slightly over a single cycle, when the channel wave propagates in each cycle can be taken as constant (it changes abruptly from cycle to cycle). Then from the condition $V(0, t) = V_0$, using (2.17), we obtain

$$Q_0 = Q(0, t) = V_0/K. \tag{3.3}$$

Relations (3.3) and (2.10) express the boundary conditions of the problem. The solution of Eqs. (2.5) and (2.9) with the conditions (3.3) and (2.10) has the form

$$\begin{gathered} Q = V_0 f(\xi)/K,\ I = V_0 g(\xi)/Z,\ E = V_0 e(\xi)/x_0(t), \\ R = Z_0 h(\xi)/x_0(t),\ \xi = x/x_0(t),\ x_0 = ct,\ c = \gamma c_0,\ Z = \beta Z_0, \end{gathered} \tag{3.4}$$

where $f(\xi)$, $g(\xi)$, $e(\xi)$ are the desired dimensionless functions for which we take $f(0) = 1$, $g(0) = 1$, $h(\xi)$ is a specified dimensionless function, Z_0 is the wave impedance (2.18), Z is the input impedance, and β and γ are parameters determined from the solution and the boundary conditions. The resistance R(x) per unit length of the channel and its dimensionless equivalent $h(\xi)$ in fact depend on the field strength of the wave. In the solution (3.4) the quantity $h(\xi)$ is considered as a functional parameter which must be matched to the actual dependence of R on the field by a rational approximation. Up to the instant $t = 0$ the resistance R is infinite since up to $t = 0$ the specific resistance of the channel in its basis is infinite. The potential of the channel charged by the previous wave after it has decayed with time decreases, and the potential difference between the cloud and the channel increases leading to an increase in the electric intensity between the cloud and the channel. At a certain instant at the initial point $x = 0$ where the axis of the channel intersects the surface of the cloud the intensity reaches a critical value E^{**} such that the intensity on the surface of the ice crystals (usually needle shaped) of the cloud or the water drops reaches the "breakdown" value E^*. Then at the initial point "local breakdown" occurs (an intense increase in the avalanche ionization) as well as a small region for which $R \neq \infty$, while the dissipative impedance Z_1 representing the Joule energy dissipation in the region is less than the wave impedance of the channel Z_0. This region is an incipient channel for the channel wave described by Eqs. (2.5) and (2.9).

Substituting (3.4) into Eqs. (2.5) and (2.9) we obtain

$$\begin{gathered} (1 - \gamma^2\xi^2) df/d\xi + hg/\beta = 0, \\ (1 - \gamma^2\xi^2) dg/d\xi + \gamma\xi hg = 0, \\ e = -df/d\xi + \gamma\xi dg/\beta d\xi = hg/\beta. \end{gathered} \tag{3.5}$$

The boundary conditions (3.3) and (2.10) with the assumed normalization $f(0) = 1$ and $g(0) = 1$, take the form (2.13). The solution of system (3.5) with (2.13) is

$$\begin{gathered} f(\xi) = \beta^{-1} \int_\xi^1 h(\xi)\, g(\xi)\, d\xi/(1 - \gamma^2\xi^2), \\ g(\xi) = \exp\left[-\int_0^\xi h(\xi)\, \xi d\xi/(1 - \gamma^2\xi^2)\right], \\ e(\xi) = \beta^{-1} h(\xi) \exp\left[-\int_0^\xi h(\xi)\, \xi d\xi/(1 - \gamma^2\xi^2)\right]. \end{gathered} \tag{3.6}$$

The values of the parameters β, γ are found from (3.6) using (2.13)

$$\beta = \int_0^1 \frac{h(\xi)\exp\left[-\int_0^{\xi} h(\xi)\,\xi d\xi/(1-\xi^2)\right]}{1-\xi^2}\,d\xi, \quad \gamma = 1. \tag{3.7}$$

The velocity of the wavefront from (3.4) and (2.7) $c = \gamma c_0 = c_0$, i.e., it is equal to the velocity of light. For the class of solutions corresponding to h = const, we find from (3.6), using (3.7),

$$f(\xi) = 1 - \frac{2}{B(1/2, h/2)}\sum_{n=0}^{\infty}\frac{(2n)!\,[1-(1-\xi^2)^{n+h/2}]}{2^{2n}(n!)(2n+h)}, \tag{3.8}$$
$$g(\xi) = (1-\xi^2)^{h/2}, \quad e(\xi) = 2(1-\xi^2)^{h/2}/B(1/2, h/2),$$

where B(p, q) is the beta function.

The parameter β in this case has the form

$$\beta = 2^{-1}hB(1/2, h/2). \tag{3.9}$$

For small $h\,(h \ll 1)$ we have from (3.9) that $\beta \simeq 1 + h \ln 2$, and for large $h\,(h \gg 1)$ we obtain from (3.9) the asymptotic formula $\beta \simeq \sqrt{\pi h/2}$. We will give the following expressions for the functions $f_h(\xi)$, $g_h(\xi)$, $e_h(\xi)$, i.e., the functions $f(\xi)$, $g(\xi)$, $e(\xi)$ from (3.6)-(3.8) for certain specific values of h, and corresponding values of β:

$$f_0 = \begin{cases} 1, & 0 \leqslant \xi < 1, \\ 0, & \xi = 1, \end{cases} \quad g_0 = \begin{cases} 1, & 0 \leqslant \xi < 1, \\ 0, & \xi = 1, \end{cases} \quad e_0 = 0, \quad \beta_0 = 1, \tag{3.10}$$
$$f_1 = 1 - 2(\arcsin\xi)/\pi, \quad g_1 = (1-\xi^2)^{1/2}, \quad e_1 = 2(1-\xi^2)^{1/2}, \quad \beta_1 = \pi/2,$$
$$f_2 = 1-\xi, \quad g_2 = 1-\xi^2, \quad e_2 = 1-\xi^2, \quad \beta_2 = 2,$$
$$f_3 = 1 - 2(\xi\sqrt{1-\xi^2} + \arcsin\xi)/\pi, \quad g_3 = (1-\xi^2)^{3/2},$$
$$e_3 = 4(1-\xi^2)^{3/2}/\pi, \quad \beta_3 = 3\pi/4,$$
$$f_4 = 1 - 3(3-\xi^2)/2, \quad g_4 = (1-\xi^2)^2, \quad e_4 = 3(1-\xi^2)^2/2, \quad \beta_4 = 8/3.$$

To describe the channel process under conditions which hinder its occurrence, we can reduce the electrical conductivity of the channel, which nevertheless simplifies the calculations. This can be achieved, in particular, by neglecting the ionizing role of the field, which penetrates into the volume of the channel, assuming that the field of the wave ionizes the air only on the surface of the channel. In this case the ionization at any point arises at the instant the surface of the channel passes through it, after which the point will be inside the volume of the channel, and a relaxation reduction in the ionization coefficient will occur in it due to recombination processes. As a result, at each point inside the channel the ionization state exists for a certain time and a corresponding electrical conductivity, which can be called a relic. One other worsening assumption is to calculate the current in the channel ignoring the part played by heating and the reduction in the density of the air in the channel particularly on its axis.

The electric field produces an electron concentration $n_1 = \chi_1/\chi_2$ in the gas, where χ_1 is the electron multiplication factor expressed in terms of the first Townsend parameter α_1 and the electron drift velocity v by the equation $\chi_1 = \alpha_1 v$, and χ_2 is the recombination coefficient [2]. If the field E_1 which produces the density n_1 momentarily disappears, the electron density begins to fall from the value $n_1 = \chi_1/\chi_2$ for the value χ_1 corresponding to the existing field, to zero by Eq. (1) given in [2] for $\chi_1 = 0$, i.e., according to the equation $dn/dt = -\chi_2 n^2$. The density n will therefore fall in accordance with the equation

$$n = n_1/(1+\chi_1 t) = 1/\chi_2(t+\tau_1), \quad \tau_1 = 1/\chi_1, \quad \chi_1 = \alpha_1 v, \tag{3.11}$$

where α_1 and v correspond to the field E_1 producing the density n_1.

Experimental values of the first Townsend coefficient α_1 as a function of the field E and the relative density of the air δ (the density of the air in units of its density at the temperature $T_0 = 273.16°K$ and a pressure $p_0 = 10^5$ N/m^2) are given approximately by the equation (with an accuracy of approximately 30%)

$$\alpha_1/\delta = A_1(E/\delta)^{1/2}\exp(-B_1\delta/E), \quad A_1 = 120\ \mathrm{V}^{-1/2}\cdot\mathrm{m}^{-1/2}, \tag{3.12}$$
$$B_1 = 1{,}5\cdot 10^7\ \mathrm{V/m}, \quad 3\cdot 10^6\ \mathrm{V/m} \leqslant E/\delta \leqslant 3\cdot 10^8\ \mathrm{V/m}.$$

The lightning usually transports negative charges [3], so that below, to be specific, we will consider negative (electron) channels. The experimental dependence of the drift velocity of the electrons on the field can be represented (with an accuracy of approximately 30%) in the form

$$v = \varkappa_1 E/\delta \text{ for } 0{,}3\cdot 10^6 \text{ V/m} \leqslant E/\delta \leqslant 3\cdot 10^6 \text{ V/m},$$
$$v = \varkappa_0 (E/\delta)^{1/2} \text{ for } 3\cdot 10^6 \text{ V/m} \leqslant E/\delta \leqslant 300\cdot 10^6 \text{ V/m},$$
$$\varkappa_1 = 0{,}057 \text{ m}^2\cdot \text{V}^{-1}\cdot \text{sec}^{-1} \text{ c}^{-1},\ \varkappa_0 = 100 \text{ m}^{3/2}\cdot \text{V}^{-1/2}\cdot \text{sec}^{-1}. \quad (3.13)$$

Values of the recombination coefficient χ_2 (determined by the dissociative mechanism) are given in [20]. The coefficient χ_2 depends on the mean electron energy, decreasing as this energy increases. The mean energy of electrons accelerated in the gas by intense ionizing fields is close to the ionization energy of the gas [21, 22]. For air the ionization potential is approximately 15 V, and the mean energy of the electrons in air when there is an intense ionizing field is close to this value. Thus, in air of normal density for fields of 10^7-10^8 V/m, which is typical for channels, the mean energy of the electrons is approximately $\varepsilon = 10$ eV. The dependence of the recombination coefficient χ_2 on the mean electron energy ε is given by the equation [20] $\chi_2 = \chi_2^0 \varepsilon^{-1.8}$, where ε is the energy in electron volts, and $\chi_2^0 = 0.8 \cdot 10^{-13}$ m³/sec. Since this value is obtained by extrapolation, it needs to be refined experimentally and below we will only use it for tentative estimates.

We introduce cylindrical coordinates r, φ, x with the x axis coinciding with the axis of the channel. Suppose r is the radial coordinate of the surface of the channel in the transverse cross section x considered, r' is the radial coordinate of the point considered inside the channel in the same cross section, r − r' is the depth of the point inside the channel, and v_0 is the mean radial velocity of the surface of the channel. The time t measured from the instant when the surface of the channel passes through the point considered, is $t = (r - r')/v_0$. Since this is the time that the point is inside the channel, the ionization at the point considered relaxes during this time, and the electron density, according to (3.11) $n = v_0/\chi_2(r - r' + d_1)$, $d_1 = 1/\alpha_1$. The current density along the channel $J = e_0 n v$, where e_0 is the electron charge, and v is the longitudinal component of the drift velocity. The longitudinal component $E = E_x$ of the electric field on the surface of the channel is much less than the transverse component E_r, and is approximately equal to the total field strength, while the longitudinal component $v = v_x$ of the drift velocity of the electrons is much less than the transverse component $v_0 = v_r$, and is approximately equal to the total velocity. Hence, the transverse component v_0 of the velocity can be expressed in terms of the transverse component E_1 of the field by (3.13), while the longitudinal component of the drift velocity $v = v_0 E/E_1$, where E is the longitudinal component of the electric field. For the current density we have (with $\delta = 1$)

$$J = \sigma E = e_0 \varkappa_0^2 E / \chi_2 (r - r' + d_1). \quad (3.14)$$

In deriving this equation we ignored capture, which becomes important when the field is reduced below 3 MV/m. On the whole, when the field falls to the critical value $E_0^* = 3$ MV/m the electrical conductivity in the channel disappears due to recombination (in a time τ_1 of the order of 10^{-8}-10^{-10} sec) due to the sharp reduction in the ionization coefficient (3.12) and as a consequence of capture (with a characteristic time of 10^{-8} sec).

The channel which occurs at the point of intersection of the axis of the channel and the surface of the cloud, propagates with the velocity of light, expanding at the drift velocity, and has a greater radius of the transverse cross section the further its cross section is from the vertex (since in this cross section the expansion process goes on for a longer time). Hence, the channel has a very elongated needle-shaped form with zero radius of the transverse cross section at the vertex $x = x_0(t)$ and a maximum at the base $x = 0$. Hence, the surface of the channel (3.1) can be approximated by a semielipsoid of rotation

$$x^2/x_0^2 + r^2/r_0^2 = 1, \quad x \geqslant 0. \quad (3.15)$$

The surface of the channel is approximated below by a diverging cone. This represents the condition of maximum mathematical simplicity for solving Maxwell's equations for a wave diverging from a point. For an infinite electrical conductivity of the channel, when there are no Joule losses in the channel, the approximation of the channel by a diverging body (as one moves away from the cloud) does not distort the propagation pattern appreciably compared with the more accurate approximation by a body of converging shape. To take into account the finite electrical conductivity a more accurate approximation is necessary, such as (3.15).

The radius of the transverse cross section of the channel at a distance x from the surface of the cloud will be represented in the form $r = r_0(t)\,\eta(\xi)$, where $\eta = r/r_0(t)$ is the dimensionless radius of the transverse cross section, and $\xi = x/x_0(t)$. For (3.15) we have

$$r = r_0(t)\eta(\xi),\ \eta(\xi) = (1 - \xi^2)^{1/2}. \quad (3.16)$$

The total current along the channel is obtained by integrating (3.14) over the whole transverse cross section of area. Multiplying (3.14) by $2\pi r' dr'$ and integrating with respect to r' from 0 to r, we obtain

$$I = E/R = 2\pi e_0 \varkappa_0^2 \Omega r E/\chi_2, \quad (3.17)$$

where $\Omega = (1 + 1/\alpha_1 r)\ \ln\ (\alpha_1 r + 1) - 1$. For field strengths of the order of 10^7-10^8 V/m, determining the ionization in the channel, and $\delta = 1$ the value of $d_1 = 1/\alpha$, from (3.12) is of the order of 10^{-6}–10^{-5} m. The radius r of the transverse cross section of the channel is of the order of a meter. Hence, $\alpha_1 r = r/d_1 \gg 1$, and in view of the logarithm, i.e., the very weak dependence of Ω on $\alpha_1 r$ the single value $\Omega = \Omega_0$ can be taken for all the transverse cross sections of the channel in the calculations, determined from the characteristic value of the radius $r = r_0$. Hence, $\Omega \simeq \Omega_0 = \ln \alpha_1 r_0$. It follows from (3.17), (3.4), (2.18), (3.15), and (3.2) that

$$h = h_0/\eta(\xi), \quad h_0 = \varepsilon_0 c_0 v_0 \chi_2 / e_0 \varkappa_0^2 \Lambda_0 \Omega_0, \quad \Omega_0 = \ln \alpha_1 x_0 / v_0. \tag{3.18}$$

The solution (3.6) for (3.18) and (3.15) takes the form

$$f(\xi) = \frac{1}{K_1(h_0)} \int\limits_{\xi}^{1} \frac{\exp[-h_0/(1-\xi^2)^{1/2}}{(1-\xi^2)^{3/2}} d\xi, \tag{3.19}$$

$$g(\xi) = \exp\left[-h_0 \frac{1-(1-\xi^2)^{1/2}}{(1-\xi^2)^{1/2}}\right], \quad e(\xi) = \frac{\exp[-h_0/(1-\xi^2)^{1/2}]}{K_1(h_0)(1-\xi^2)^{1/2}},$$

where K_1 is MacDonald's function. For $h_0 \ll 1$ $\beta = 1 + h_0 + 0.5 h_0^2 \ln h_0/2 + 0.03681 h_0^2$ with $h_0 \gg 1$ $\beta \simeq \sqrt{\pi h_0/2}$. In (3.19), as in (3.10), together with (2.13), $e(1) = 0$, but solutions are possible in which $e(1) \neq 0$.

The charges in the channel, in view of high electrical conductivity, are concentrated in the surface layer. The surface charge density ω can be expressed in terms of the charge Q per unit length of the channel by the equation $\omega = Q/2\pi r$. From the boundary conditions the intensity E_1 on the surface of the channel will be $E_1 = \omega/\varepsilon_0 = Q/2\pi\varepsilon_0 r$. Using (3.4), (2.6), and (2.16) we obtain

$$E_1 = fE_0 r_0/r = fE_0/\eta, \; E_0 = V_0/\Lambda_0 r_0 = v_0 V_0/\Lambda_0 x_0 \tag{3.20}$$

(E_0 is the field strength at the base of the channel $x = 0$), where, according to (2.13), (3.4), (2.15), and (2.16) $f = 1$ and $\eta = 1$. We always have $\eta \leq 1$ and usually $f \simeq 1$ over the whole length of the channel, apart from an extremely narrow region in the front. The field strength (3.20) considerably exceeds the critical value $E_0^* = 3$ MV/m. Thus, for $V_0 = 10^8$ V [3, 12], $\nu_0 \simeq 10^3$ (which, as can be seen, in fact occurs), $\Lambda_0 = \ln 2\nu_0 = 7.6$ for $x_0 = 10^3$ m we have the least value of the field strength on the surface of the channel $E_0 = 1.3 \cdot 10^7$ V m, which was already obtained above using Maxwell's equations for channels of conical shape. Over almost the whole length of the channel we usually have $f \simeq 1$, and from (3.20) we obtain $E_1 > E_0$. A typical range of field strengths E_1 on the surface of the channel is in the range from 10^7 to 10^8 V/m. For the calculated field strength of $1.3 \cdot 10^7$ V/m the characteristic ionization decay time after the field is switched off according to (3.11) $\tau_1 = 1/\chi_1 = 1/\alpha_1 v$ taking (3.12) and (2.13) into account is $0.2 \cdot 10^{-10}$ sec, for a characteristic channel length $x_0 = 1$ km the parameter Ω_0 in (3.18) in equal to 12, and in view of its logarithmic nature it depends only slightly (when $\alpha_1 r_0 \gg 1$) on the specific conditions.

The "self-ionization" of the channels together with the electromagnetic speed of propagation of the channel waves leads to the observed long length of lightning. The expansion of the channel with the drift velocity while the channel wave propagates with the velocity of light over kilometer distances does not enable the field on the surface of the channel to fall below the critical value of $E_0^* = 3$ MV/m, for which the ionization process ceases and the electrical coupling with the cloud as a source of lightning energy disappears. Nevertheless the field strength (3.20) is very much connected with the electromagnetic nature of the process. At high supplying potentials, when the propagation is strongly electromagnetic in nature and occurs quasiideally ($\beta \simeq 1$), the values of Q_0 and I_0, i.e., the values of Q and I at the base of the channel, are connected by the equation $Q_0 = I_0/c_0$, as follows from (3.4) and (2.13) or from (2.19). The existence of fairly intense currents equal to for quasiideal propagation according to (2.19), (3.4), and (2.13) $I_0 \simeq V_0/Z_0$, are connected with the values Q_0 $I_0/c_0 = V_0/c_0 Z_0$, which lead, taking (2.18) into account, to high field strengths E_0 according to (3.20). In the electrostatic picture for a good conducting streamer limited by the surface (3.15) and adjoining the well-conducting surface of the electrode, using the well-known solution of the electrostatic problem on a conducting ellipsoid in an external field [19] it can be shown that $E_0 = 0$. The field strength on the surface of the streamer in the region of the electrode is found to be less than the critical value $E_0^* = 3$ MV/m, and there is no electrical conductivity in the streamer on the surface of the electrode. The streamer is disconnected, isolated from the electrode, and cannot obtain energy from the source. Hence, the streamers in air of normal density only develop in fairly intense external fields of the order of 3 MV/m or greater, when they can acquire the energy necessary to develop directly from the external field. In the electromagnetic picture of the relaxation theory the energy is transported along the channel of the discharge in the form of an electromagnetic flux from a very high-power source (the cloud), situated at the base of the channel. This, however, only occurs during the stage of electromagnetic activity. At the initial instants of the calm stages electrostatic equilibrium of the channel and the cloud occurs, the electric field on the surface of the channel and at its base, i.e., at the boundary with the cloud, vanishes, and the electrical conductivity at the base of the channel disappears. At the initial instants of the calm stages, the channel is therefore electrically switched off from the cloud and develops during the calm stages autonomously without obtaining energy and charge from the cloud until the channel wave of the next cycle occurs.

The radius r_0 of the base of the channel increases as given by the equation $dr_0/dt = v$, where the velocity v is found from (3.13) for $E = E_0$ given by (3.20). The channel occurs at the point $x = 0$, $r = 0$, so that $r_0(0) = 0$. Since the channel propagates with the velocity of light c_0, its vertex $x = x_0$ reaches the vertex of the channel $x = a_0$ at the instant $t_0 = a_0/c_0$. At this instant the radius $r_0(t)$ of the base of the channel (where its transverse cross section is a maximum) reaches its highest value $b_0 = r_0(t_0)$. The average velocity of expansion of the channel at its base $v_0 = b_0/t_0 = b_0 c_0/a_0$. Hence, when $\delta = 1$, taking (3.2) into account, we have

$$b_0 = \left(9\varkappa_0 a_0^2 V_0 / 4c_0^2 \Lambda_0\right)^{1/3}, \quad v_0 = \left(9c_0 \varkappa_0^2 V_0 / 4\Lambda_0 a_0\right)^{1/3} = c_0/\nu_0. \tag{3.21}$$

When solving the "longitudinal" problem the transverse expansion of the channel at its base is assumed, according to (3.2), to occur with a constant velocity of v_0 in accordance with (3.21).

When the channel wave propagates the radius r_0 of the base of the channel increases with time, while the field strength E_0 on the surface of the channel at the base $x = 0$, in accordance with (3.20), decreases. At the instant when the field strength E_0 reaches the critical value $E_0^* = 3$ MV/m, the electrical conductivity in the part of the channel close to the cloud disappears (in a time of the order of 10^{-8}–10^{-10} sec). The lightning channel is electrically disconnected, insulated from the cloud, and the flow of energy from the cloud ceases. In this case the channel wave, generally speaking, may propagate further for a certain distance as a rectangular electromagnetic pulse of finite length along the waveguide. This propagation is not supported by the source, and the channel wave decays rapidly, using the stored energy reserve. We will assume for simplicity that the propagation of the channel wave ceases immediately after the channel is disconnected from the cloud. Since the field strength on the surface of the base of the channel is, according to (3.20), $E_0 = V_0/\Lambda_0 r_0$, the greatest possible radius $r_0 = b_0^*$, determined by the condition $V_0/\Lambda_0 b_0^* = E_0^*$, will be the maximum radius of the base of the greatest channel (the channel of the last cycle), while the length $a_0 = a_0^*$ of this channel is given by (3.21). Putting $a_0^* = l$, $b_0^* = s$, and taking (3.2) into account we have

$$l = 2c_0 V_0 / 3\Lambda_0 \varkappa_0 \left(E_0^*\right)^{3/2}, \quad s = V_0/\Lambda_0 E_0^*, \tag{3.22}$$

$$\nu_0 = 2c_0 / 3\varkappa_0 \left(E_0^*\right)^{3/2}, \quad \Lambda_0 = \ln 2\nu_0.$$

Since the value of l is the greatest length of the channel in all the cycles it is the length of the lightning. The value of s, i.e., the greatest radius of the base of the channel in all the cycles, is the maximum radius of luminosity of the lightning. Substituting into (3.22) the value $c_0 = 3 \cdot 10^8$ m/sec, and $E_0^* = 3 \cdot 10^6$ V/m and from (3.13) $\varkappa_0 = 10^2\ \mathrm{m}^{3/2} \cdot \mathrm{V}^{-1/2} \cdot \mathrm{sec}^{-1}$, we find from (3.22) $\nu_0 = 1150$ and $\Lambda_0 = 7.74$.

For l and s we obtain the expressions $l = 5.0 \cdot 10^{-5}\ V_0$m, and $s = 4.3 \cdot 10^{-8}\ V_0$m. The potentials V_0 of the storm clouds lie in the range from $3 \cdot 10^7$ to 10^{10} V [3, 12, 5, 15, 7].

For the least potential of $3 \cdot 10^7$ V the length of lightning is found to be 1.5 km and the radius of the luminosity is 1.3 m. The experimental minimum length of the lightning is 2 km [3] and 1 km [12], and the minimum observed radius of the luminosity of the propagating lightning is 0.5 m [3]. For the most probable value of the cloud potential of 10^8 V [3, 12] the theoretical length of the lightning is 5 km, and the radius of the luminosity is 4.3 m. The experimentally most probable ("characteristic") length of the lightning is 5 km [3, 5, 7]. The radii of the luminosity of the lightning lie in the range from 0.5 m to 5 m, which is also the range given by calculation. For a cloud potential of $3 \cdot 10^8$ V [5] the length of the lightning is 15 km, and according to experimental data is 14 km [5]. The theoretical value of the maximum radius of the luminosity in this case is 13 m, but there is no corresponding experimental data. Such longer lengths of lightning are usually horizontal, and develop inside the clouds where it is difficult to measure the parameters of the discharge channel. For a cloud potential of $4 \cdot 10^9$ V [15] the theoretical length of the lightning is 200 km. The greatest length which has in fact been observed is 150–160 km [11, 6]. The value of the potential of 10^{10} V [3] is in fact a nonrealizable upper limit, and therefore lightning with a corresponding length of 500 km does not occur. Thus, the relaxation theory determines lightning lengths in agreement with observational results. Note that according to streamer theory the length of the lightning should be 0.2 km [10], which is 25 times less than the experimental characteristic length of 5 km [3, 5, 7], and 750 times less than the experimental maximum length of the lightning of 150 km [11, 6].

When a channel wave propagates conditions are possible in which high electrical conductivity produced in the channel by the field of the wave behind its front occurs not immediately behind the front but at a certain distance from it. This occurs when the electric field strength of the wave is comparatively small and reaches a value sufficient for avalanche ionization only at a certain distance from the front. Then at the point where the high electrical conductivity occurs, a second, in this case the main, front occurs. The electrical conductivity of the channel in the region between the fronts — the region between the leading and main fronts — is connected with the initial bare ionization of the channel and is extremely small. Although the relative energy losses of the field of the wave in the region between the fronts is high, the main flow of energy occurs behind the main front and with respect to the total energy of the field of the wave the losses in the region between the fronts is negligibly small. Hence, the energy losses in the channel wave are determined by dissipation in the region behind the main front, and if the relative energy losses there are small, propagation occurs almost ideally with the velocity of the main front, very close to the speed of light. Note that according to the Sommerfeld–Brillouin theorem [19] the leading fronts of the electromagnetic excitations propagate with the fundamental velocity (the velocity of light in a vacuum) irrespective of the properties of the medium, while the leading front must usually be close to the main front, the velocity of which depends on the specific propagation conditions.

The propagation of the channel wave with the leading and main fronts can be described most simply using a piecewise-constant function $h(\xi)$ in (3.4). In the region $0 \le \xi \le \xi_1$ behind the main front $\xi = \xi_1$ the field produces high electrical conductivity characterized by the quantity $h = h_0$, while in the region between the fronts $\xi_1 < \xi \le 1$, where the electrical conductivity corresponds to the initial bare ionization and is extremely small, $h = h_1 \gg h_0$. Since the function $h(\xi)$ is always finite, $h \ne \infty$, from (3.6) and (2.13) it follows that $\gamma = 1$, i.e., the leading front $x_0(t)$ according to (3.4) propagates with the velocity of light, $x_0 = c_0 t$. The main front $x_1(t)$ propagates in this case according to (3.4) with a velocity $c_1 = \xi_0 c_0$, $x_1 = c_1 t = \xi_1 c_0 t$. The parameter ξ_1 is found using the solution (3.6) and the additional condition expressing the fact that the intensity E_1 on the surface of the channel defined by (3.20) on the main front $x = x_1$, $\xi = \xi_1$, where $r = r_1$, is equal to the critical field strength $E_0^* = 3$ MV/m, for which high electrical conductivity is produced in the channel

$$f(\xi_1)\,\eta^{-1}(\xi_1)\,E_0 = E_0^*. \tag{3.23}$$

From the solution (3.6) for the conditions when $h_0/h_1 \ll 1$, $1 - \xi_1 \ll 1$, we have

$$f(\xi_1) = \beta^{-1}(1-\xi_1)^{h_0/2}, \quad \beta = 2^{-1} h_0 B(1/2, h_0/2), \tag{3.24}$$

where $B(p, q)$ is the beta function. In the case of a cylindrical channel $r = r_0$ we have $r_1 = r_0$, $\eta(\xi_1) = 1$, and for the conditions when $1 - \xi_1 \ll 1$, using (3.23) and (3.24) we find the velocity $c_1 = \xi_1 c_0$ of the main front

$$c_1 = \left[1 - 2^{-1}(\beta E_0^*/E_0)^{2/h_0}\right] c_0. \tag{3.25}$$

Using the values determined above, namely, $\nu_0 = 1.150 \cdot 10^3$, $\Lambda_0 = 7.74$, $\Omega_0 = 12$, $\chi_2 = 10^{-15}$ m³/sec, and $\varkappa_0 = 100$ m$^{3/2}$/V$^{1/2}$ · sec and taking $\delta = 1$, we find from (3.18) $h_0 = 0.02$, and $\beta = 1.02 \simeq 1$, and from (3.25) $c_1 = [1 - 2^{-1}(E_0^*/E_0)^{100}]c_0$. Since $E_0^* = 3$ MV/m and from the estimate given above $E_0 = 13$ MV/m, we have $c_1 \simeq (1 - 2 \cdot 10^{-64})c_0$, i.e., the leading and main fronts practically coincide. In practice, the channel has a form which is expanding from the head to the base, i.e., approximately the form given by (3.15). At the points $0 \le \xi \le \xi_1$ according to (3.18) and (3.16) $h = h_0/(1 - \xi^2)^{1/2}$, and for $\xi > \xi_1$ the value of h is determined by the initial small ionization of the charged channel (the part of the channel in front of the wavefront), along which propagation occurs. If when $0 \leqslant \xi \leqslant \xi_1$ $h = h_0/(1-\xi^2)^{1/2}$, and for $\xi > \xi_1$ for the value of h we have $h < h_1 \equiv h_0/(1-\xi_1^2)^{1/2}$, and the main front moves more rapidly than in the case when $h = h_0/(1 - \xi^2)^{1/2}$ over the whole range $0 \le \xi \le 1$. In fact, along the ideal line ($h = 0$) the main front, like the leading front, propagates with the velocity of light, and for $h \ne 0$ as h is reduced the velocity of propagation increases. Hence, a lower estimate of the velocity of the main front when $h < h_1$ can be obtained using (3.23) with (3.16) and (3.19).

According to (3.5) and $dg = \beta\gamma\xi df$, and, since $\gamma = 1$ and for $h_0 \ll 1$ $\beta \simeq 1$, taking (2.13) into account we have for the neighborhood of the point $\xi = 1.1 - \xi \ll 1$, $f \simeq g$. Equation (3.23) has the form

$$\eta(\xi_1) = k_0 g(\xi_1), \tag{3.26}$$

where $k_0 = E_0/E_0^*$. Hence, using (3.16) and (3.19) for $h_0 \ll 1$ we obtain the equation for ξ_1 : $\eta_1 = \ln k_0 \eta_1^{-1} = h_0$, $\eta_1 = \eta(\xi_1)$ from (3.16). For $h_0 = 0.02$, $E_0 = 13$ MV/m, and $E_0^* = 3$ MV/m we have $\xi_1 = 0.999996$. Hence, the velocity of the main front $c_1 = \xi_1 c_0$ is almost identical with the velocity of the leading front, i.e., the velocity of light.

The solutions considered describe the propagation of locally fed channel waves when there is no external field. In practice they include the usual cases for lightning when the part played by the external field in the ionization of the gas in the channel is negligibly small compared with the part played by the inherent field of the propagating wave. If the external field has a considerable influence on the ionization of the gas in the channel the propagation conditions are improved. The inherent field of the channel wave on the surface of the channel is almost normal to this surface, since the longitudinal component is small compared with the transverse component. Inside the channel the transverse component is negligibly small since it is related to the charges of the channel, which are concentrated on its surface. Hence, inside the channel the main component is the longitudinal component of the electric field of the wave, i.e., the electric field of the channel wave inside the channel is longitudinal (parallel to the exit of the channel). This field is usually small and insufficient to ionize the gas inside the channel. If there is a longitudinal external field approximately equal to the critical value $E_0^* = 3$ MV/m, the inherent longitudinal field of the wave, added to the external field, leads to intense ionization over the whole volume of the channel, which produces its high electrical conductivity. Unlike the relic electrical conductivity produced by the surface field and concentrated on the surface of the channel, which occurs when there is an external field, the "active" electrical conductivity has a volume character. In this connection, when describing the propagation of the channel wave in intense external fields, to simplify the calculations the relic electrical conductivity can be ignored. The actual conditions of propagation of the channel waves are favorable, since the energy losses in the channel waves are less the greater the electrical conductivity occuring in the channels.

Suppose that a field with an electric intensity E due to ionization arrives in a certain volume of the gas. At the same time intense recombination occurs. As a result, in unit volume of the gas an electron number density is established given by [2] $n_1 = \chi_1/\chi_2$, where $\chi_1 = \alpha_1 v$ is the electron multiplication factor, and α_1 and v are given by (3.12) and (3.13). The current density is $J = e_0 n_1 v = e_0 \alpha_1 v^2/\chi_2$. Using (3.12) and (3.13) and the value given above of $\chi_2 = 10^{-15}$ m³/sec we obtain for the current density

$$J/\delta = C_1(E/\delta)^{3/2} \exp(-B_1\delta/E), \quad C_1 = 192 \text{ C} \cdot \text{sec}^{-1} \cdot \text{V}^{-3/2} \text{m}^{-3/2}. \tag{3.27}$$

For mathematical simplicity this relation can be approximated by the approximate equation

$$J = \sigma(E - E^*)^2 = \sigma_0 \delta^{-1}(E - E_0^* \delta)^2, \tag{3.28}$$

$$\sigma_0 = 2{,}6 \cdot 10^{-2} \text{ A/V}^2, \; E_0^* = 3 \cdot 10^6 \text{ V/m},$$

where, when $E < E^*$ the current density is assumed to be zero. This equation in the most important range of E/δ from $3 \cdot 10^6$ V/m to $3 \cdot 10^7$ V/m gives too low a value of the current density determined from (3.27) while in the range from $5 \cdot 10^6$ V/m to $3 \cdot 10^7$ V/m it reproduces it with an accuracy to within 10%. (Similar equations for the current density in [1, 2] are based on erroneous initial data.)

The most interesting and simple case mathematically is when the external field is parallel to the axis of the channel and has a value equal to the critical value E^*, corresponding to the beginning of local breakdown (an intensely increasing avalanche ionization at the point considered). In this case the total longitudinal electric field will be $E' = E^* + E$, where E is the longitudinal field of the wave. Then, substituting into (3.28) for E the quantity $E' = E^* + E$ (where the quantity E is now related to the field of the wave) we obtain

$$J = \sigma E^2 = \sigma_0 \delta^{-1} E^2, \; \sigma_0 = 2{,}6 \cdot 10^{-2} \text{ A/V}^2. \tag{3.29}$$

Suppose $S = S(x) = \pi r^2(x)$ is the transverse cross section of the channel at a distance x from the cloud. Then in terms of this transverse cross section $I = JS$, in which case from (3.29) we have $E = (I/\sigma S)^{1/2}$. Denoting further by E and I the corresponding values, taking their signs into account (and not their absolute value as above), and using (3.16), we obtain

$$E = I(\pi r^2 \sigma |I|)^{1/2}. \tag{3.30}$$

The system (2.5) and (3.30), substituting (3.4) and taking (3.16) into account, reduces to the equations

$$(1 - \gamma^2\xi^2)\eta\, df/d\xi + \sqrt{2g/\alpha\beta} = 0, \tag{3.31}$$

$$(1 - \gamma^2\xi^2)\eta\, dg/d\xi + \beta\gamma\xi\sqrt{2g/\alpha\beta} = 0,$$

$$\beta\eta e = \gamma\eta\xi\, dg/d\xi - \beta\eta\, df/d\xi = \sqrt{2\beta g/\alpha}.$$

The boundary conditions (3.3) and (2.10) with the assumed normalization $f(0) = 1$, $g(0) = 1$ take the form (2.13). The channel parameter α in (3.31) has the form

$$\alpha = \sigma_0 \Lambda_0 V_0 / \varepsilon_0 c_0 v_0^2 \delta. \tag{3.32}$$

The solution of (3.31) with (2.13) can be found in the form

$$f(\xi) = \alpha^{-1} \int_\xi^1 G(\xi)\, d\xi/(1 - \gamma^2\xi^2), \quad g(\xi) = \beta G^2(\xi)/2\alpha, \tag{3.33}$$

$$e(\xi) = G(\xi)/\alpha\eta(\xi), \; G(\xi) = \gamma \int_\xi^1 \xi\, d\xi/(1 - \gamma^2\xi^2)\,\eta(\xi),$$

where the dependence of the parameters β and γ on α is given by the equations

$$\beta = 2\alpha G^2(0), \quad \int_0^1 G(\xi)\, d\xi/(1 - \gamma^2\xi^2)\,\eta(\xi) = \alpha. \tag{3.34}$$

The solution (3.33) and (3.34) for an eliptic channel (3.15) is given in [1]. For $\alpha \ll 1$ this solution has the form $f(\xi) = 1 - \xi$, $g(\xi) = 1 - \xi^2$, $e(\xi) = 1$, and the dependence of the parameters β, and γ on α is given by the expansions $\beta = 2\alpha + 10\alpha/9 + \ldots$, $\gamma = \alpha - 10\alpha^3/9 + \ldots$. When $\alpha \ll 1$, which corresponds to lightning conditions, the solution has the form $f(\xi) \simeq 1$, $g(\xi) \simeq 1$, $e(\xi) \ll 1$ for all ξ, besides the narrow region near the front of width $\Delta\xi = 1 - \gamma = \pi^2/16\alpha$, where $f(\xi)$ and $g(\xi)$ fall rapidly to zero on the front, and $e(\xi)$ increases rapidly to a value on the front of $e(1) = 8\pi^2$. The parameters β and γ can be expressed in terms of the asymptotic formulas

$$\beta \simeq 1 + \sqrt{2/\alpha},\ \gamma \simeq \sqrt{8\alpha/(8\alpha + \pi^2)} \simeq 1 - \pi^2/16\alpha.$$

Using the values obtained above for $\sigma_0 = 0.026$ A V^2, $\nu_0 = 1150$, and $\Lambda_0 = 7.74$ for $V_0 = 10^8$ V we obtain from (3.32) $\alpha = 5730$. The values of β and γ are found to be $\beta = 1.02$ and $\gamma = 0.9999$. The front $x_0 = \gamma c_0 t$ is in this case the main front, since the field strength on the front exceeds the critical value $E_0^* = 3$ MV/m. In fact, the external field strength under the conditions assumed in this case is equal to E_0^* and addition of the wave field leads to intense ionization immediately behind the front, where the field of the wave has a finite value. There is no leading front under the conditions considered since we have ignored the relic and bare electrical conductivity. The velocity of the main front $c = \gamma c_0 = 0.9999 c_0$ hardly differs from the velocity of light c_0.

Since the characteristic impedance Z_0 is, by definition, the input impedance of an ideal line, the channel behaves with respect to the channel wave of specified power more ideally the closer the input impedance Z is to the characteristic impedance Z_0. Hence, the condition for the channel to be ideal like a waveguide line for channel waves transferring the lightning energy, is

$$Z' \equiv Z - Z_0 \ll Z_0. \tag{3.35}$$

It can be shown that the value of Z' differs from the dissipative impedance Z_1 determined above, which determines the Joule losses, only by a factor of the order of unity, and the quantity $\zeta = Z'/Z_0 \simeq Z_1/Z_0$ expresses (for $Z' \ll Z_0$) the fraction of the energy lost by the channel wave as it propagates. The condition (3.35) for the propagation to be ideal taking (3.4) into account can be written in the form

$$D \equiv Z_0/(Z - Z_0) = 1/(\beta - 1) \gg 1. \tag{3.36}$$

The lost fraction of the energy is $\zeta = 1/D$. The quantity D which expresses by the condition $D \to \infty$ the ideal nature of the propagation, is similar to the selectivity. The usual idea of selectivity relates to linear systems. A lightning channel is not such a system, since its electrical conductivity depends on the field of the propagating wave. However, the value of D from (3.36) represents the departure of the process from ideal, and as $D \to \infty$ it defines propagation without loss. In this sense the quantity D can be regarded as the selectivity, which, however, depends not only on the properties of the channel but also on the power of the propagating wave. From (3.36) for large D for the solutions (3.4), (3.19), and (3.4) and (3.33) for (3.15) and (3.16) we have the following expressions: $D = 1/h_0$ and $D = \sqrt{\alpha/2}$. Using the calculated values of h_0 and α for $V_0 = 10^8$ V, $h_0 = 0.02$, and $\alpha = 5730$ we obtain D = 50 and 54 respectively.

Note that when a considerable amount of energy is transmitted along the lightning channel we have $D \approx 1$, since in this case the losses are of the order as the transmitted energy, and greater amounts of energy of the order of the electric energy of the cloud may reach the earth.

LITERATURE CITED

1. B. N. Kozlov, "Principles of the relaxation theory of ball lightning," Dokl. Akad. Nauk SSSR, 221, No. 4, 802 (1975); 224, No. 1, 10 (1975).
2. B. N. Kozlov, "The theory of ball lightning," Fiz. Plazmy, 4, No. 1, 159 (1978).
3. M. Yuman, Lightning [Russian translation], Mir, Moscow (1972).
4. B. J. Mason, Physics of Clouds, Oxford Univ. Press (1972).
5. A. F'yu, "Thunder," Usp. Fiz. Nauk, 119, No. 4, 735 (1976).
6. B. Mason, "Charge generation in storms," in: Problems of Atmospheric Electricity [in Russian], Gidrometeoizdat, Leningrad (1969).
7. S. Singer, The Nature of Ball Lightning, Plenum (1971).
8. J. Meek and J. Craggs, Electric Breakdown in Gases [Russian translation], IL, Moscow (1960).
9. É. D. Lozanskii and O. B. Firsov, "Qualitative theory of a streamer," Zh. Eksp. Teor. Fiz., 56, No. 2, 670 (1969).
10. É. D. Lozanskii and O. B. Firsov, Theory of the Spark [in Russian], Atomizdat, Moscow (1975).
11. I. M. Imyanitov, "Lightning," BSE, 16, 468 (1974).
12. B. F. Schonland, Flight of Thunderbolts, Oxford Univ. Press (1964).
13. I. M. Shagin, Storms on the Black Sea (Photographs), Ogonek, No. 20 (1960); Priroda No. 9, 84 (1966).
14. M. T. Dmitriev, "Ball lightning: new observations and new hypotheses," Priroda, No. 6, 50 (1971).
15. C. W. Allen, Astrophysical Quantities, Humanities (1973).
16. I. S. Stekol'nikov, Storms on the Black Sea (Explanations of Photographs by I. M. Shagin), Ogonek, No. 20, 32 (1960).

17. B. N. Kozlov, "Maximum energy dissipation of ball lightning," Dokl. Akad. Nauk SSSR, 238, No. 1, 4.
18. I. E. Tamm, Principles of the Theory of Electricity [in Russian], Nauka, Moscow (1976).
19. J. A. Stratton, Electromagnetic Theory, McGraw-Hill (1941).
20. D. H. Douglas-Hamilton, "Recombination rate measurements in nitrogen," J. Chem. Phys., 58, No. 11, 4820 (1973).
21. H. Schlumbohm, "Electronenlawinen bei hohen E/P," Z. Phys., 182, No. 3, 306 (1965).
22. C. S. Lacshminarasimha and J. Lucas, "The ratio of the radial diffusion coefficient to the mobility for electrons in helium, argon, air, methane, and nitric oxide," J. Phys. D: Appl. Phys., 10, No. 3, 313 (1977).